INTERNATIONAL MATHEMATICAL OLYMPIAD
"FORMULO DE INTEGRECO", 2012/13

PROBLEMS OF THE SECOND ROUND FOR SENIOR PARTICIPANTS

1. Consider the number 123456789. It is allowed to take 2 digits of the same parity and replace each of them with their arithmetic mean. Is it possible to receive a number greater than 800000000 using this operation several times?

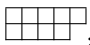
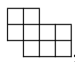
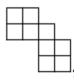
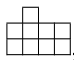
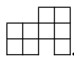
2. The sum of three distinct positive integers is equal to 2013. Find the greatest possible value for their GCD (greatest common divisor).

3. The points X and Y are taken on the sides AB and BC of a triangle ABC. AY and CX intersect each other at the point Z. It is known that $AY = YC$ и $AB = CZ$. Prove that the points B, Y, Z, X lie on a circumference.

4. Prove the inequality

$$(1 + x + 2x^2)(2 + 3y + y^2)(4 - 11z + 8z^2) \geq 7xyz$$

for any positive numbers x, y and z.

5. There are n plane pieces in a puzzle, each of them consists of 9 squares and has one of the following shapes: , , , , or . Prove that for any odd n it is impossible to construct a rectangle $9 \times n$ from these pieces.