INTERNATIONAL MATHEMATICAL OLYMPIAD "FORMULO DE INTEGRECO", 2012

PROBLEMS OF THE FIRST ROUND FOR JUNIOR PARTICIPANTS

1. Is it possible to draw 4 triangles in such a way that each triangle contains exactly one vertex of each of three other triangles? Vertices of any triangle cannot belong to the sides of other triangles.

2. Children thought of a natural number and made several statements: "The number is divisible by 2", "The number is divisible by 3 but is not divisible by 2"; "The number is divisible by 4 but is not divisible by 3"; \ldots ; "The number is divisible by 10 but is not divisible by 9". What maximal number of the statements can be true?

3. Peter runs a round track. Each 5 minutes he runs past Maria who is rocking on swing, and each 15 minutes he outruns elderly Michael who is also running in the same direction. At a certain moment, Peter turned around and started to run in the opposite direction. How often does he meet Michael now?

4. Can the numbers $1, 2, \ldots, 2012$ be divided into pairs in such a way that in each pair the sum of numbers contains only digits 0 and 4 in its decimal representation?

5. There are checkers in several squares of 8×8 board (not more than one checker in a square). It is known that there are 4 or more checkers in each row and in each column. Is it always possible to remove some checkers in order to receive exactly 4 checkers in each row and in each column?

6. There are 15 elephants standing in a row, each elephant weighs an integer number of kilograms. For each elephant (except for the last one), the sum of its weight and doubled weight of the next elephant equals 15000 kg. Find the weight of each elephant (and prove that you have found all possible variants).